# SIMULATION OF INTERSECTING BLACK BRANE SOLUTIONS BY MULTI-COMPONENT ANISOTROPIC FLUID

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A family of spherically symmetric solutions with horizon in the model with multi-component anisotropic fluid (MCAF) is obtained. The metric of any solution contains $(n-1)$ Ricci-flat "internal space" metrics and for certain equations of state ($p_i = \pm\rho$) coincides with the metric of intersecting black brane solution in the model with antisymmetric forms. Examples of simulation of intersecting $M2$ and $M5$ black branes are considered. The post-Newtonian parameters $\beta$ and $\gamma$ corresponding to the 4-dimensional section of the metric are calculated.

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## 1 Introduction

Recently, spherically-symmetric $p$-brane solutions with horizon (see, e.g., [1] and references therein) defined on product manifolds $\mathbf{R} \times M_0 \times \ldots \times M_n$ cause a wide interest. These solutions appear in models with antisymmetric forms and scalar fields. These and more general $p$-brane cosmological and spherically symmetric solutions are usually obtained by reduction of the field equations to the Lagrange equations corresponding to Toda-like systems [4]. An analogous reduction for models with multi-component anisotropic fluids was performed earlier in [6]. For cosmological-type models with antisymmetric forms without scalar fields any $p$-brane is equivalent to an anisotropic fluid with the equations of state:

$$\hat{p}_i = -\hat{\rho} \qquad \text{or} \qquad \hat{p}_i = \hat{\rho}, \tag{1.1}$$

when the manifold $M_i$ belongs or does not belong to the brane world volume, respectively (here $\hat{p}_i$ is the effective pressure in $M_i$ and $\hat{\rho}$ is the effective density).

In this paper we find the analogues of intersecting black brane solutions in a model with multi-component anisotropic fluid (MCAF), when certain "orthogonality" relations on fluid parameters are imposed. The one-component case was considered earlier in [12].

The paper is organized as follows. In Section 2 the model is formulated. In Section 3 general MCAF solutions with horizon corresponding to black-brane-type solutions are presented. Section 4 deals with certain MCAF analogues of intersecting black brane solutions, i.e. $M2$ and $M5$ black brane solutions. In Section 5 the post-Newtonian parameters for the 4-dimensional section of the MCAF-black-brane metric are calculated. In Appendix based on [1, 7] the general spherically symmetric solutions with multicomponent anisotropic fluid are considered and configurations with horizon are singled out.

## 2 The model

In this paper we consider a family of spherically symmetric solutions to Einstein equations with an anisotropic matter source

$$R^M_N - \frac{1}{2}\delta^M_N R = kT^M_N, \tag{2.1}$$

defined on the manifold

$$M = \underset{\substack{\text{radial}\\ \text{variable}}}{\mathbf{R}_*} \times \underset{\substack{\text{spherical}\\ \text{variables}}}{(M_0 = S^{d_0})} \times \underset{\text{time}}{(M_1 = \mathbf{R})} \times \ldots \times M_n, \tag{2.2}$$

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with the block-diagonal metrics

$$ds^2 = e^{2\gamma(u)}du^2 + \sum_{i=0}^{n} e^{2X^i(u)} h^{[i]}_{m_i n_i} dy^{m_i} dy^{n_i}. \tag{2.3}$$

Here $\mathbf{R}_* \subseteq \mathbf{R}$ is an open interval. The manifold $M_i$ with the metric $h^{[i]}$, $i = 1, 2, \ldots, n$, is a Ricci-flat space of dimension $d_i$:

$$R_{m_i n_i}[h^{[i]}] = 0, \tag{2.4}$$

and $h^{[0]}$ is the standard metric on the unit sphere $S^{d_0}$, so that

$$R_{m_0 n_0}[h^{[0]}] = (d_0 - 1) h^{[0]}_{m_0 n_0}; \tag{2.5}$$

$u$ is a radial variable, $\kappa$ is the gravitational constant, $d_1 = 1$ and $h^{[1]} = -dt \otimes dt$.

The energy-momentum tensor is adopted in the following form for each component of the fluid:

$$(T^{(s)}{}^M_N) = \mathrm{diag}(-\hat\rho^{(s)}, \hat p^{(s)}_0 \delta^{m_0}_{k_0}, \hat p^{(s)}_1 \delta^{m_1}_{k_1}, \ldots, \hat p^{(s)}_n \delta^{m_n}_{k_n}), \tag{2.6}$$

where $\hat\rho^{(s)}$ and $\hat p^{(s)}_i$ are the effective density and pressures respectively, depending on the radial variable $u$.

We assume that the following "conservation laws"

$$\nabla_M T^{(s)M}_N = 0 \tag{2.7}$$

are valid for all components.

We also impose the following equations of state

$$\hat p^{(s)}_i = \left(1 - \frac{2U^{(s)}_i}{d_i}\right) \hat\rho^{(s)}, \tag{2.8}$$

where $U^{(s)}_i$ are constants, $i = 0, 1, \ldots, n$.

The physical density and pressures are related to the effective ones (with "hats") by the formulae

$$\rho^{(s)} = -\hat p^{(s)}_1, \quad p^{(s)}_u = -\hat\rho^{(s)}, \quad p^{(s)}_i = \hat p^{(s)}_i \quad (i \neq 1). \tag{2.9}$$

In what follows we put $\kappa = 1$ for simplicity.

# 3 Spherically symmetric solutions with horizon

We will make the following assumptions:

$$\begin{aligned}
&1^o. \quad U^{(s)}_0 = 0 \ \Leftrightarrow\ \hat p^{(s)}_0 = \hat\rho^{(s)}, \\
&2^o. \quad U^{(s)}_1 = 1 \ \Leftrightarrow\ \hat p^{(s)}_1 = -\hat\rho^{(s)}, \\
&3^o. \quad (U^{(s)}, U^{(s)}) = U^{(s)}_i G^{ij} U^{(s)}_j > 0, \quad (U^{(s)}, U^{(l)}) = 0, \quad s \neq l,
\end{aligned} \tag{3.1}$$

where

$$G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D}, \tag{3.2}$$

are components of the matrix inverse to the matrix of the minisuperspace metric [5]

$$(G_{ij}) = (d_i \delta_{ij} - d_i d_j), \tag{3.3}$$

and $D = 1 + \sum_{i=0}^{n} d_i$ is the total dimension.

The orthogonality condition $3^o$ is an integrability condition (see Appendix). The conditions $1^o$ and $2^o$ in p-brane terms mean that brane "lives" in a time manifold $M_1$ and does not "live" in $\mathbf{R}_* \times M_0$. The assumptions $1^o$ and $2^o$ are natural ones from the point of view of state equations (2.8), so we can rewrite the energy-momentum tensor (2.6) as following:

$$(T^{(s)}{}^M_N) = \mathrm{diag}(-\rho^{(s)},\ \rho^{(s)} \delta^{m_0}_{k_0},\ -\rho^{(s)} \delta^{m_1}_{k_1},\ p^{(s)}_2 \delta^{m_2}_{k_2},\ \ldots, p^{(s)}_n \delta^{m_n}_{k_n}). \tag{3.4}$$

Under the conditions (2.8) and (3.1) we have obtained the following black-hole solutions to the Einstein equations (2.1):

$$ds^2 = J_0 \left( \frac{dr^2}{1 - 2\mu/r^d} + r^2 d\Omega^2_{d_0} \right) - J_1 \left( 1 - \frac{2\mu}{r^d} \right) dt^2 + \sum_{i=2}^{n} J_i h^{[i]}_{m_i n_i} dy^{m_i} dy^{n_i}, \tag{3.5}$$

$$\rho^{(s)} = - \frac{A_s}{H_s^2 J_0 r^{2d_0}}, \qquad A_s = -\frac{1}{2}\, \nu_s^2 d^2 P_s (P_s + 2\mu), \tag{3.6}$$

which may be verified from [6] and by analogy with the $p$-brane solution [4]. For direct derivation of the solution see Appendix. Here $d = d_0 - 1$,

$$d\Omega^2_{d_0} = h^{[0]}_{m_0 n_0} dy^{m_0} dy^{n_0} \tag{3.7}$$

is the spherical element,

$$J_i = \prod_{s=1}^{m} H_s^{-2\nu_s^2 U^{(s)i}}, \qquad H_s = 1 + P_s / r^d; \tag{3.8}$$

$P_s > 0,\ \mu > 0$ are integration constants and

$$U^{(s)i} = G^{ij} U^{(s)}_j = \frac{U^{(s)}_i}{d_i} + \frac{1}{2 - D} \sum_{j=0}^{n} U^{(s)}_j, \tag{3.9}$$

$$\nu_s = (U^{(s)}, U^{(s)})^{-1/2}. \tag{3.10}$$

# 4 Simulation of intersecting black branes

The solution from the previous section for MCAF allows to simulate the intersecting black brane solutions [1] in the model with antisymmetric forms without scalar fields. In this case the parameters $U^{(s)}_i$ have the following form:

$$\begin{array}{cccccl} U^{(s)}_i & = & d_i, & p^{(s)}_i & = & -\rho^{(s)}, \quad i \in I_{(s)}; \\ & & 0, & & & \rho^{(s)}, \quad i \notin I_{(s)}. \end{array} \tag{4.1}$$

Here $I_{(s)} = \{i_1, \ldots, i_k\} \in \{1, \ldots n\}$ is the index set [1] corresponding to brane submanifold $M_{i_1} \times \ldots \times M_{i_k}$.

The orthogonality constraints $3^o$ (3.1) lead us to the following dimension of intersection of brane submanifolds [1]:

$$d_{I_{(s)} \cap I_{(l)}} = \frac{d_{I_{(s)}} d_{I_{(l)}}}{D - 2}, \tag{4.2}$$

where $d_{I_{(s)}}$ and $d_{I_{(l)}}$ are dimensions of $p$-brane world-volumes, $s, l = 1, \ldots, m,\ s \neq l$.

Due to relations (4.1) and $1^o$, (3.1) we can rewrite (3.6) as follows:

$$\rho^{(s)} = - \frac{A_s}{H_s^2 \prod\limits_{l=1}^{m} H_l^{2/(D - d_{I_{(l)}} - 2)} r^{2d_0}}, \tag{4.3}$$

and investigate the behavior of the density as a radial function. For the single fluid the density is regular and positive at zero when the parameter $d$ (see the previous section) is equal to $d^* = D - d_{I_{(1)}} - 2$. In this case the brane submanifold fills the total manifold (2.2) except $R_* \times S^{d_0}$.When $d < d^*$ the density is infinite at zero.

For multi-component fluid all densities are finite at $r = 0$, if (and only if)

$$\sum_{s=1}^{m} \frac{1}{D - d_{I_s} - 2} \geq \frac{1}{d}. \tag{4.4}$$

Moreover, all $\rho^{(s)}(0) > 0$ when the equality in (4.4) takes place.

As an example we consider simulation by MCAF of intersecting $M2 \cap M5$, $M2 \cap M2$, $M5 \cap M5$ configurations in $D = 11$ supergravity. The metric for all cases reads:

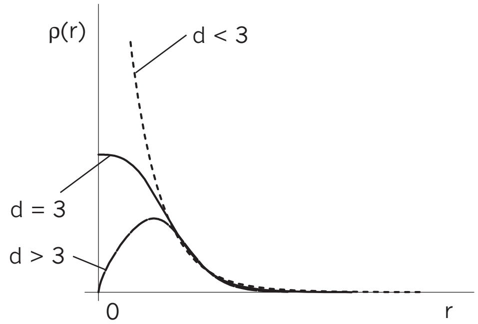


Figure 1: The variants of behavior of $\rho(r)$ for $M2 \cap M2$ intersection

$$ds^2 = J_0 \left[ \frac{dr^2}{1-2\mu/r} + r^2 d\Omega^2_{d_0} - (H_{(I)})^{-1}(H_{(II)})^{-1} \left\{ \left(1 - \frac{2\mu}{r}\right) dt^2 + h^{[2]}_{m_2 n_2} dy^{m_2} dy^{n_2} \right\} \right.$$
$$\left. + H^{-1}_{(I)} h^{[3]}_{m_3 n_3} dy^{m_3} dy^{n_3} + H^{-1}_{(II)} h^{[4]}_{m_4 n_4} dy^{m_4} dy^{n_4} + h^{[5]}_{m_5 n_5} dy^{m_5} dy^{n_5} \right], \tag{4.5}$$

where we can express the factor $J_0 = H_{(I)}^{2/(D-d_{I_{(I)}}-2)} H_{(II)}^{2/(D-d_{I_{(II)}}-2)}$; the first brane world-volume is $M_1 \times M_2 \times M_3$, the second one is $M_1 \times M_2 \times M_4$.

*a*). For MCAF, corresponding to intersecting of $M2$ (with index $I$ in (4.5)) and $M5$ (with index $II$) branes the dimensions are following $d_1 = d_2 = d_3 = 1,\ d_4 = 4$ and $J_0 = H^{1/3}_{(I)} H^{2/3}_{(II)}$.

The densities $\rho^{(I)},\ \rho^{(II)}$ are infinite at zero when $d = 1$ and for $d = 2$ they are finite: $\rho^{(I)}(0) = (P_I + 2\mu)/H_I^{4/3} H_{II}^{2/3},\ \rho^{(II)}(0) = (P_{II} + 2\mu)/H_{II}^{5/3} H_I^{1/3}$. It is interesting to note that in the extremal limit $\mu \to 0$ $\rho^{(I)}(0) = \rho^{(II)}(0)$.

*b*). For MCAF equivalent to two electrical $M2$ branes intersecting on the time manifold we get $d_3 = d_4 = 2,\ d_2 = 0$. Here $J_0 = H^{1/3}_{(I)} H^{1/3}_{(II)}$.

The variants of behavior of the densities are presented on Figure 1. When $d = 3$ both functions are regular and positive at zero (the middle branch).

*c*). For two $M5$ branes the dimension of intersection is 4 and $d_0 = d_3 = d_4 = 2,\ d_2 = 3,\ d_5 = 0$ and $J_0 = H^{2/3}_{(I)} H^{2/3}_{(II)}$. The only possibility here is $d = 1$ and the fluid densities are infinite at zero.

# 5 Physical parameters

## 5.1 Gravitational mass and post-Newtonian parameters

Here for simplicity we put $d_0 = 2$ ($d = 1$). Consider the 4-dimensional space-time section of the metric (3.5). Introducing a new radial variable by the relation

$$r = R\left(1 + \frac{\mu}{2R}\right)^2, \tag{5.1}$$

we rewrite the 4-section in the following form:

$$ds^2_{(4)} = g^{(4)}_{\mu\mu'} dx^\mu dx^{\mu'} = \left(\prod_{s=1}^{m} H_s^{-2\nu_s^2 U^{(s)0}}\right) \left[ -\left(\frac{1-\mu/2R}{1+\mu/2R}\right)^2 \left(\prod_{s=1}^{m} H_s^{-2\nu_s^2}\right) dt^2 + \left(1 + \frac{\mu}{2R}\right)^4 \delta_{ij} dx^i dx^j \right], \tag{5.2}$$

$i, j = 1,2,3$. Here $R^2 = \delta_{ij} x^i x^j$.

The post-Newtonian (Eddington) parameters are defined by the well-known relations

$$g^{(4)}_{00} = -(1 - 2V + 2\beta V^2) + O(V^3), \tag{5.3}$$
$$g^{(4)}_{ij} = \delta_{ij}(1 + 2\gamma V) + O(V^2), \tag{5.4}$$

$i,j=1,2,3$. Here $V=GM/R$ is the Newtonian potential, $M$ is the gravitational mass and $G$ is the gravitational constant. From (5.2)-(5.4) we obtain:

$$GM=\mu+\sum_{s=1}^{m}\nu_s^2 P_s(1+U^{(s)0}), \tag{5.5}$$

and

$$\beta-1 \;=\; \frac{1}{2(GM)^2}\sum_{s=1}^{m}\;\nu_s^2 P_s(P_s+2\mu)(1+U^{(s)0}), \tag{5.6}$$

$$\gamma-1 \;=\; -\frac{1}{GM}\sum_{s=1}^{m}\;\nu_s^2 P_s(1+2U^{(s)0}). \tag{5.7}$$

For fixed vector $U^{(s)}$ the parameter $\beta-1$ is proportional to the ratio of two physical parameters: the anisotropic fluid density parameter $A_s$ (see (B.15)), and the gravitational radius squared $(GM)^2$.

### 5.2 The Hawking temperature

The Hawking temperature of a black hole may be calculated using the relation from [8] and has the following form:

$$T_H=\frac{d}{4\pi(2\mu)^{1/d}}\prod_{s=1}^{m}\left(\frac{2\mu}{2\mu+P_s}\right)^{\nu_s^2}. \tag{5.8}$$

## 6 Conclusions

Here we have obtained a family of spherically symmetric solutions with horizon in the model with multi-component anisotropic fluid with the equations of state (2.8) and the conditions (3.1) imposed. The metric of any solution contains $(n-1)$ Ricci-flat "internal" space metrics. For certain equations of state (with $p_i=\pm\rho$) the metric of the solution may coincide with the metric of intersecting black branes (in a model with antisymmetric forms without dilatons). Here the examples of simulating of intersecting $M2$ and $M5$ black branes in $D=11$ supergravity are considered.

We have also calculated the post-Newtonian parameters $\beta$ and $\gamma$ corresponding to the 4-dimensional section of the metric. The parameter $\beta-1$ is written in terms of ratios of the physical parameters: the anisotropic fluid parameter $|A_s|$ and the gravitational radius squared $(GM)^2$. An open problem is to generalize the formalism to the case when dilaton scalar fields are added into consideration.

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## Appendix

### A Lagrange representation

The "conservation law" equation (2.7) may be written, due to relations (2.3) and (2.6) in the following form:

$$\dot{\hat{\rho}}^{(s)}+\sum_{i=0}^{n}d_i\dot{X}^i(\hat{\rho}^{(s)}+\hat{p}_i^{(s)})=0. \tag{A.1}$$

Using the equation of state (2.8) we get

$$\hat{\rho}^{(s)}=-A_s e^{2U_i^{(s)}X^i-2\gamma_0}, \tag{A.2}$$

where $\gamma_0(X)=\sum\limits_{i=0}^{n}d_iX^i$, and $A_s$ are constants.

The Einstein equations (2.1) with the relations (2.8) and (A.2) imposed are equivalent to the Lagrange equations for the Lagrangian

$$L = \frac{1}{2} e^{-\gamma + \gamma_0(X)} G_{ij} \dot{X}^i \dot{X}^j - e^{\gamma - \gamma_0(X)} V, \tag{A.3}$$

where

$$V = \frac{1}{2} d_0(d_0 - 1) e^{2U_i^{(0)} X^i} + \sum_{s=1}^{m} A_s e^{2U_i^{(s)} X^i} = \sum_{s=0}^{m} A_s e^{2U_i^{(s)} X^i}, \tag{A.4}$$

is the potential and the components of the minisupermetric $G_{ij}$ are defined in (3.3).

$$U_i^{(0)} X^i = -X^0 + \gamma_0(X), \qquad U_i^{(0)} = -\delta_i^0 + d_i, \qquad A_0 = \frac{1}{2} d_0(d_0 - 1), \tag{A.5}$$

$i = 0, \ldots, n$.

For $\gamma = \gamma_0(X)$, i.e. when the harmonic time gauge is considered, we get the set of Lagrange equations for the Lagrangian

$$L = \frac{1}{2} G_{ij} \dot{X}^i \dot{X}^j - V, \tag{A.6}$$

with the zero-energy constraint imposed

$$E = \frac{1}{2} G_{ij} \dot{X}^i \dot{X}^j + V = 0. \tag{A.7}$$

It follows from the restriction $U_0^{(s)} = 0$ that

$$(U^{(0)}, U^{(s)}) \equiv U_i^{(0)} G^{ij} U_j^{(s)} = 0. \tag{A.8}$$

Indeed, the contravariant components $U^{(0)i} = G^{ij} U_j^{(0)}$ are the following ones

$$U^{(0)i} = -\frac{\delta_0^i}{d_0}. \tag{A.9}$$

Then we get $(U^{(0)}, U^{(s)}) = U^{(0)i} U_i^{(s)} = -U_0^{(s)}/d_0 = 0$. In what follows we also use the formula

$$\frac{1}{\nu_0^2} = (U^{(0)}, U^{(0)}) = \frac{1}{d_0} - 1 < 0, \tag{A.10}$$

for $d_0 > 1$.

In what follows we will make the following assumption on indices: $s = 1, \ldots, m$ and $\alpha = 0, \ldots, m$.

# B General spherically symmetric and cosmological-type solutions

When the orthogonality relations (A.8) and $3^o$ of (3.1) are satisfied the Euler-Lagrange equations for the Lagrangian (A.6) with the potential (A.4) have the following solutions (see relations from [7] adopted for our case):

$$X^i(u) = -\sum_{\alpha=0}^{m} \frac{U^{(\alpha)i}}{(U^{(\alpha)}, U^{(\alpha)})} \ln |f_\alpha(u - u_\alpha)| + c^i u + \bar{c}^i, \tag{B.1}$$

where $u_\alpha$ ($\alpha = 0, \ldots, m$) are integration constants; and vectors $c = (c^i)$ and $\bar{c} = (\bar{c}^i)$ are orthogonal to the $U^{(\alpha)} = (U^{(\alpha)i})$, i.e. they satisfy the linear constraint relations

$$U^{(0)}(c) = U_i^{(0)} c^i = -c^0 + \sum_{j=0}^{n} d_j c^j = 0, \tag{B.2}$$

$$U^{(0)}(\bar{c}) = U_i^{(0)} \bar{c}^i = -\bar{c}^0 + \sum_{j=0}^{n} d_j \bar{c}^j = 0, \tag{B.3}$$

$$U^{(s)}(c) = U_i^{(s)} c^i = 0, \tag{B.4}$$

$$U^{(s)}(\bar{c}) = U_i^{(s)} \bar{c}^i = 0. \tag{B.5}$$

Here

$$f_\alpha(\tau) = \begin{array}{ll} R_\alpha \dfrac{\mathrm{sh}(\sqrt{C_\alpha}\tau)}{\sqrt{C_\alpha}}, & C_\alpha \neq 0, \quad \eta_\alpha = +1, \\ R_\alpha \dfrac{\mathrm{ch}(\sqrt{C_\alpha}\tau)}{\sqrt{C_\alpha}}, & C_\alpha > 0, \quad \eta_\alpha = -1, \\ R_\alpha \tau, & C_\alpha = 0, \quad \eta_\alpha = +1, \end{array} \qquad \text{(B.6)}$$

$R_\alpha = \sqrt{2|A_\alpha/\nu_\alpha^2|}$, $\eta_\alpha = -\,\mathrm{sign}(A_\alpha/\nu_\alpha^2)$; and parameters $\nu_\alpha$ are defined in (3.10) and (A.10), $\alpha = 0, \ldots, m$.

The zero-energy constraint, corresponding to the solution (B.1) reads

$$E = \frac{1}{2} \sum_{\alpha=0}^{m} \frac{C_\alpha}{(U^{(\alpha)}, U^{(\alpha)})} + \frac{1}{2} G_{ij} c^i c^j = 0. \qquad \text{(B.7)}$$

From (B.1) we get the following relation for the metric (see also (3.3), (A.9) and (A.10))

$$g = \mathrm{e}^{2c^0 u + 2\bar{c}^0} \left( \prod_{\alpha=0}^{m} f_\alpha^{-2\nu^2 U^{(\alpha)0}} \right) \left\{ du \otimes du + f_0^2 \; h^{[0]} \right\} + \sum_{i \neq 0} \mathrm{e}^{2c^i u + 2\bar{c}^i} \left( \prod_{\alpha=0}^{m} f_\alpha^{-2\nu^2 U^{(\alpha)i}} \right) h^{[i]}, \qquad \text{(B.8)}$$

where $f_\alpha = f_\alpha(u - u_\alpha)$ (here we use the relations $d_i U^i + \frac{U_0}{d_0} = U^0$ and (A.10)).

**Solutions with horizon.** For integration constants we put

$$\bar{c}^i = 0, \qquad \text{(B.9)}$$

$$c^i = \bar{\mu} \sum_{\alpha=0}^{m} \frac{U^{(\alpha)i}}{(U^{(\alpha)}, U^{(\alpha)})} - \bar{\mu}\delta_1^i, \qquad \text{(B.10)}$$

$$C_\alpha = \bar{\mu}^2, \qquad \text{(B.11)}$$

where $\bar{\mu} > 0$, $\alpha = 0, \ldots, m$.

We also introduce new radial variable $r = r(u)$ by relations

$$\exp(-2\bar{\mu}u) = 1 - \frac{2\mu}{r^d}, \quad \mu = \bar{\mu}/d > 0, \quad d = d_0 - 1, \qquad \text{(B.12)}$$

and put $u_0 = 0$; $u_s < 0$, $A_s < 0$, $s = 1, \ldots, m$,

$$\frac{\sqrt{2|A_s|}}{\bar{\mu}\nu_s} \,\mathrm{sh}\, \beta_s = 1, \quad \beta_s \equiv \bar{\mu}|u_s|. \qquad \text{(B.13)}$$

Now the parameter $P_s$ may be introduced ($P_s > 0$) by the following relation:

$$\frac{\mu}{\mathrm{sh}\, \beta_s} = P_s e^{\beta_s} = \sqrt{P_s(P_s + 2\mu)}, \qquad \text{(B.14)}$$

and, hence,

$$-A_s = \frac{1}{2} \nu_s^2 d^2 P_s (P_s + 2\mu), \qquad \text{(B.15)}$$

see (A.2). The relations of the Appendix imply the formulae (3.5), (3.6) for the solution from Section 3.

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## List of captions for illustrations

Figure 1. The variants of behavior of $\rho(r)$ for $M2 \cap M2$ intersection.